4. Tables of infrared absorption frequencies often list the absorption frequencies for carbon-carbon single, double and triple bonds with ranges around 1200 cm⁻¹, 1660 cm⁻¹ and 2180 cm⁻¹ respectively.

a. The mass of a carbon atom is $1.99 \times 10^{-26}$ kg. What is the reduced mass, $\mu$, of a pair of carbon atoms?

The reduced mass, $\mu$, of two masses, $m_1$ and $m_2$, is defined as:

$$\frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2}$$

If the two masses are equal, this becomes:

$$\mu = \frac{m_1}{2}$$

So, for 2 carbon atoms, the reduced mass is:

$$\mu = \frac{1.99 \times 10^{-26}}{2} = 1.00 \times 10^{-26} \text{ kg}$$

b. Force constants, $k$, for the stretching motion between carbon atoms joined by single, double and triple bonds are given in the table below. Calculate the predicted absorption frequencies of the three types of bonding situations. How do these compare with the frequencies given in Table 5-2 of SUPL-005?

<table>
<thead>
<tr>
<th>Bond Type</th>
<th>$k$ (Newton/m = kg·sec⁻²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C --- C</td>
<td>500</td>
</tr>
<tr>
<td>C == C</td>
<td>1000</td>
</tr>
<tr>
<td>C ≡≡ C</td>
<td>1500</td>
</tr>
</tbody>
</table>

$$\nu = \frac{1}{2\pi} \left( \frac{k}{\mu} \right)^{1/2} = 0.1592 \left( \frac{k}{1 \times 10^{-26}} \right) = 1.592 \times 10^{12} \times k^{1/2}$$

Which gives: $35.7 \times 10^{13}$, $50.3 \times 10^{13}$ and $61.7 \times 10^{13}$ for the frequencies of the three bond types. To calculate the frequencies in wave numbers (cm⁻¹) we need the reciprocals of the corresponding wavelengths which are given by:

$$\frac{1}{\lambda} = \nu / c$$ (where $c$ is the velocity of light in cm/sec, namely, $2.997 \times 10^{10}$ cm/sec)

The calculations produce 1191, 1678 and 2059 cm⁻¹ respectively for the single, double and triple bonded carbon atoms.

These numbers compare very favorably with the values listed in SUPL-005 which are 1200, 1680 and 2180 respectively.