PHY 123 Lab 9 - Poiseuille's Law

The purpose of this lab is to study the flow of water in long, thin (capillary) tubes under conditions satisfying Poiseuille's Law. (Pronunciation of his name: Pwah-zoy; when used as an adjective: Pwah-zellian.) The assertion (Sutera and Skalak, Annu. Rev. Fluid Mech. 1993. 25:1-19) that "Poiseuille's law is one of the few equations derived from applied mechanics that is well known in the present medical community." should motivate students in this course to pay close attention to this lab. This law applies to the venous flow of blood in your body, which is in the "laminar" regime of low-velocity flow. At high velocity fluid flow becomes "turbulent", a complicated regime this experiment mostly avoids. Your body doesn't avoid it. Some arterial blood flow, particularly in diseased and stenotic (narrowed) arteries, can be turbulent and produce detectable acoustical noise, e.g., "murmurs". The "whooshing" sound heard with a stethoscope in blood pressure measurement with an inflatable cuff and sphygmomanometer comes from turbulence. Between laminar and turbulent flow is a "transition regime" that this experiment will reveal.

To prepare for this experiment you should review Ch. 13 Fluids in KJF2 [Knight, Jones and Field, College Physics: A Strategic Approach, 2nd ed.], our textbook for this course. This material was covered in lecture and homework last week, 10-15 April. For this Lab pay special attention to Ch. 13.7, Viscosity and Poiseuille's Equation: the text, figures, Examples, and Integrated Example 13.15 on p. 430.

Important! You need to print out the worksheet package you find by clicking on this link [http://www.ic.sunysb.edu/Class/phy121pk/labs/pdfs/Poiseuilles Law worksheet Lab 9 PHY 123 Spring 2012.pdf] and bring it with you to your lab session.

If you need the .pdf version of these instructions you can get them here [http://www.ic.sunysb.edu/Class/phy121pk/labs/pdfs/Poiseuilles Law Lab 9 PHY 123 Spring 2012.pdf].

Introduction

"Fluids” – things that “flow” – are all around us and inside us. The simplest is a Newtonian fluid [http://en.wikipedia.org/wiki/Newtonian_fluid], which flows like a fluid no matter how force is applied to it. A non-Newtonian fluid [http://en.wikipedia.org/wiki/Non-Newtonian_fluid] is not so simple. When forced "slowly", it flows like a liquid. When forced "rapidly", it behaves more like a solid. (See this [watch it all!] [http://www.youtube.com/watch?v=f2XQ97XHjVw] and other videos for entertaining applications of these properties.) Even solids can flow, e.g., dry granular media such as sand, rice, and powders flowing through openings much larger than the characteristic size of the solid grains.

One can visualize flow patterns of transparent fluids if they are “seeded” with small “objects” that follow it with minimal perturbation and can scatter light. A famous example (see Fig. 13.22 on p. 420 of KJF2 or go here [http://www.physlink.com/education/askexperts/Images/ae464a.jpg]) is smoke rising from a smoldering tip that heats the air above it, causing it to become less dense and rise. Minute smoke particles in the rising air seed the flow and allow us to see its developing patterns. Close to the smoldering tip, where the flow velocity is relatively slow, the vertical smoke column rises smoothly in the "laminar" flow regime. Farther up, where the flow velocity increases above a rather abrupt threshold, the flow starts to circulate in corkscrew-like “vortices”. This is the “transition region”. A bit farther up, at even higher velocity, the flow becomes very confused with vortices and more complicated structures rapidly proliferating. This is the “turbulent regime”.

Other seeded flows, particularly for water and other wet fluids, use tiny gas bubbles or injected dye jets to visualize the flow patterns. You'll see examples of both of these in the three-clip video below.

The resistance to flow is lower for laminar flow and higher when turbulence kicks in. This is of enormous practical and economic importance because "resistance" means that energy must be expended to maintain the flow. Higher flow resistance means higher power is needed to maintain the flow, and that expenditure of energy doesn't come for free. Have you wondered why most modern cars have roughly the same
teardrop shape in the front and why an increasing percentage of commercial jetliners have “bent wingtips”? The answer is that these features tend to minimize turbulent drag from the motion through the air. Result: lower fuel consumption.

It’s not surprising, then, that evolution has produced a circulatory system using predominantly laminar flow in healthy individuals. This minimizes the expenditure of energy and allows the heart to work more efficiently and last longer.

Who was Poiseuille? What is his Law?

Jean Leonard Marie Poiseuille (1979-1869) entered the Ecole Polytechnique in Paris, then, as now, the pre-eminent French applied science and engineering school, in 1815, but because of political turmoil in Paris, the school was closed for a while in April, 1816. During his brief time there Poiseuille was already revealing his penchant for precise, careful experimentation, a trait which he continued to develop during his entire career. He switched to studying medicine (which, it seems, he never practiced) and submitted his doctoral thesis in 1828 on Studies of the force of the aortic heart. His most celebrated paper, completed in 1842 and published in 1846, is Experimental studies on the movement of liquids in tubes of very small diameter. Though he did not derive it theoretically, he showed experimentally the flow through such tubes is proportional to $K \frac{\Delta P D^4}{L}$, where $K$ is a proportionality constant that depends on the particular fluid, its temperature, etc., and $\Delta P$ is the pressure drop across a tube of length $L$ and diameter $D$. Similar, but less precise experimental results, were obtained a bit earlier by Gotthilf Heinrich Ludwig Hagen in Berlin, who published his first paper on the flow of water in cylindrical tubes in 1839. At about 11'40” into the three-clip video, below, the narrator uses an experimental apparatus based on what Hagen used. Theoretical derivations of Poiseuille’s Law began to appear in in the late 1850s.

Using the notation in Ch. 13 of KJF2, see Eq. (13.17), Poiseuille’s Law for the (laminar) flow rate $Q$ [units: m$^3$/s = volume per unit time) of a viscous, incompressible fluid can be written

$$Q = \frac{\pi R^4 \Delta P}{8 \eta L},$$

(9.1)

where $v_{avg}$ is the average speed (units: m/s) of the fluid in the tube of circular cross sectional area $A$ (units: m$^2$), $R$ (unit: m) is the tube radius, $\Delta P$ (unit: Pa=N/m$^2$) is the pressure drop along the length $L$ (unit: m) of the tube, and $\eta$ [units: Pa-s=kg/(m·s)] is the “dynamic” (or “absolute”) viscosity of the fluid. (Table 13.3 on p. 428 of KJF2 has viscosities for a few fluids at different temperatures.) For use below, the “kinematic viscosity” [units: m$^2$/s] is defined by $\nu = \frac{\eta}{\rho}$, where $\rho$ is the fluid density [units: kg/m$^3$]. In our discussion below $\Delta P$ will be the pressure drop between the “ingress” end and “egress” end of the capillary flow-tube-under-test.

For water at 20 °C, $\rho = 10^3$ kg/m$^3$, $\eta = 10^{-3}$ Pa·s, and $\nu = 10^{-6}$ m$^2$/s.
Video (clips extracted from 3 of a series of 25 educational films on fluid mechanics made in the early 1960s)

Viewed as a whole that lasts 20'22”, all three clips provide an excellent introduction to the difference between laminar flow, for which Poiseuille's Law is applicable, and turbulent flow. The first clip comes from here [http://www.youtube.com/watch?v=hALx7vfmRt4]. The narrator, Geoffrey Ingram Taylor, now deceased, is one of the scholarly giants of fluid mechanics. The second clip, which begins at 7'51” on the white time bar, comes from here [http://www.youtube.com/watch?v=oOGXEfgKttM]. The third clip, which begins at 17'43” on the white time bar, comes from here [http://www.youtube.com/watch?v=eIHv3cIujU&feature=related]. Follow the links if you want to see more such material.

Equipment

- one gravity-driven, water flow apparatus with two different capillary tubes
- computer workstation for data collection
- The KMPlayer free software for watching and timing videos continuously or frame-by-frame
- semilog graph paper (two log 10 cycles by 52-box linear) with the worksheet for this lab
- clear plastic ruler with 12 inch scale and 30 cm scale

Figure 1 shows how the experiment works. Note that we have only one such apparatus because water flowing in breakable glassware on tables having computers nearby is troublesome. Video technology, however, allows students at each lab station independently to collect and analyze data from two pre-recorded videos. The experiment is relatively simple. Water is poured through the funnel to fill the annular region between the two cylinders. If one of the two capillary tubes is coiled up below the cylinders, the water flows through it and drains the annular region. A video camera records how the water meniscus in that annular region falls with time. Because the “hydrostatic pressure head” that drives the flow decreases
As the meniscus falls, the initial flow is much faster than the flow later on. You will certainly notice this in the videos.
One video, called OrangeTubeRunDscf3296.flv and located on the desktop of your computer in the lab room, was recorded for water flow through the “orange tube”; see Fig. 2 just above here. Notice the still picture shows that the flow is a steady stream. The relatively rapid flow means that this “orange-tube run” lasts about 16 minutes. However, because the flow behavior at the beginning of the run is perturbed by turbulence, you need to collect data from the whole video. As you will see when you plot your data from this run, only the last part of the run has laminar flow that is governed by Poiseuille’s Law, Eq. (9.1). It is from that later part of the run that you will extract the data you need to make comparisons to the “whitish tube” run.

The other video, called WhitishTubeRunDscf3306.flv and also located on the desktop of your computer in the lab room, was recorded for water flow through the "whitish tube"; see Fig. 3 just above here. Notice that the still picture shows that the drip-by-drip flow through this tube is much slower than it is for the orange tube. This explains why the video for it lasts 57'20". However, to get enough data for this “whitish-tube run” you need only the first 42 minutes of so of the video. If you’re running out of time in the lab, even the first 20 minutes of this run will suffice because all of this run is in the laminar flow regime governed by Poiseuille’s Law, Eq. (9.1).

6-minute video on using The KMPlayer for this experiment

Since The KMPlayer (the same freeware you used for Lab 4: Speed of a Bullet) not only counts frames but also keeps track of elapsed time, you can use it for precise and accurate time measurements in this experiment. Play the video just below here now. You will need to use it to answer one (or more) of the questions in the online pretest for this Lab. While you watch/use this video, make sure you realize that the controls for The KMPlayer you see are only in the video. You cannot use them here! The only control you can use to pause, move forward, or move backward in the video you’re watching here are those in the white timebar at the bottom. When you do the experiment in the A-121 laboratory room, you will be using the actual KMPlayer to control your watching of the two pre-recorded videos of the water flow.
Some details about the apparatus

Figure 1 gives most of the quantitative details about the apparatus, so we need only add some more words and then move to deriving some equations that are needed.

Water poured into the funnel will fill the annular region between the two cylinders. The region is open at the top. A rubber stopper closes off the bottom of the graduated cylinder except for two capillary tubes passing upward through the stopper and a few mm into the space above. Each capillary has the same length but different inner diameters. We give you the inner diameter of the “orange” tube but not the inner diameter of the “whitish” tube. If both tubes are left (coiled up) at the bottom of the apparatus, water will flow out through them, emptying the annular region between the two cylinders. If the ends of both tubes are raised above the initial water level in the annular region, no water will flow. Therefore, to do a controlled flow experiment, we leave one tube (the “non-flow” tube) raised above the initial water level in
the annular region, and we put the other tube (the flow-tube-under-test) coiled up on top of the red plastic disk near the bottom of the graduated cylinder.

Because of the relatively large inner diameter of the graduated cylinder, it would hold A LOT OF WATER unless we use the volume-reducing cylinder to limit this. Without that volume-reducing cylinder, it would take A LONG TIME for the flow-tube-under-test to empty the cylinder. Because you will need this to calculate the actual water flow rates for the online pre-test and other uses, you will need to calculate the cross-sectional area of the annular region between the tubes. Knowing the inner diameter of the graduated cylinder and the outer diameter of the volume-reducing cylinder, this is easy.

We arrange two other details: (1) that the water “ingress” and water “egress” ends of whichever tube is the flow-tube-under-test are at the same height. This simplifies the calculation of the pressure difference across the flow-tube-under-test; cf., Eq. (9.4). (2) As you can see in Figs. 2 and 3, the egress water flows into a puddle on the upper surface of the red plastic disk. This (further) minimizes any “inertial effects” by slowing the egress water velocity just as it leaves the flow-tube-under-test. This slowing occurs because the effective area of the flow in the puddle is much larger than it is inside the flow-tube-under-test; cf., Eq. (9.2).

Using Poiseuille's Law with other formulae

The equation of continuity, Eq. (13.12) on p. 420 of KJF2, is an extension of the first of the equalities in Eq. (9.1) above:

$$Q = v_1 A_1 = v_2 A_2$$

(9.2)

where the subscripts “1” and “2” refer to different locations along the flow, and we drop the “avg” subscript to simplify notation. Let’s take location “1” to be somewhere in the annular region between the two cylinders in Fig. 1 and location “2” to be somewhere in the capillary flow-tube-under-test. First apply Eq. (9.2) to location “1”: $Q = A_1 v_1$.

You may easily calculate annular area $A_1$ because you know the outer diameter of the flow-reducing cylinder and the inner diameter of the graduated cylinder? Put those values on your worksheet, and then calculate $A_1$ by twice using the formula for the area of a circle followed by a simple subtraction. Look carefully at Figs. 1 and 5 to realize that $A_1$ is the same for any height above the 4.2 cm line on the graduated cylinder. Since you will not be collecting any flow data below the 5 cm line, $A_1$ will have the same value, which you must calculate, for your whole experiment.

The speed $v_1$, however, is not constant, as simple viewing of the pre-recorded videos for each flow-tube-under-test will show. Moreover, we expect it not to be constant because as the vertical column of water is drained via flow through the capillary tube below it, the height of the meniscus drops. This decreases the hydrostatic pressure $pgh$, that drives the flow. The pre-recorded video of the flow allows us to measure $v_1$ by measuring $\frac{dh}{dt}$. The discussion below shows that if certain conditions are met, we expect the decrease of $h$ to be exponential in time. You will find that the flow through the whitish tube meets the conditions and “gives a good exponential decay” in time.

So now write

$$Q = -A_1 \frac{dh}{dt}$$

(9.3)
The minus sign is needed because the height $h$ of the vertical water column decreases in time.

The flow through the capillary flow-tube-under-test is “driven” by the pressure difference between its two ends. Its “ingress” end (see Figs. 4 and 5) is at the 2.1 cm line on the graduated cylinder.

Figure 4 is just below here.

Figure 5 is just below here.
Therefore, the pressure there, \(P\), is 1 atmosphere (because the top of the vertical tube is open to the air) plus the hydrostatic pressure \(\rho gh\) of the vertical water column, where \(h\) is the height of the meniscus above the 2.1 cm line. Since the egress end is open to the air, the pressure \(P_e\) at the “egress” end is 1 atmosphere. Moreover, since its “egress” end and “ingress” end are at the same height, there is no difference-in-height-caused additional hydrostatic pressure difference for that tube.

Using \(\Delta P\) for \(P_i - P_e\), the pressure difference across the flow-tube-under-test is

\[
\Delta P = \rho gh
\]

(9.4)

Combining all the equations above, we get

\[-A_1 \frac{dh}{dt} = \frac{\pi \rho g R_i^4}{8 \eta L} h.\]

(9.5)

Rewrite this as

\[
\frac{dh}{dt} = -\lambda h,
\]

(9.6)

where

\[
\lambda = \frac{\pi \rho g R_i^4}{8 \eta L A_1}
\]

(9.7)

has [units: 1/s]. \(\lambda\) is the time constant for a temporal decay. Integrating Eq. (9.6) gives

\[
h = h_0 e^{-\lambda t},
\]

(9.8)

where \(h_0\) is the height at \(t = 0\), which should be the beginning of the flow measurement, which we’ll take to be when the meniscus crosses the 100 cm line on the graduated cylinder. All values of \(h(t)\), including \(h_0\), must be corrected to give the height above the 2.1 cm line of the graduated cylinder:

\[
h_0 = (100 - 2.1) \text{ cm} = 97.9 \text{ cm}.
\]

(9.9)
Eq. (9.8) is the equation for exponential decay promised above. (If this was new to you, it no longer is, and you will see it again at least twice next semester in PHY 122/124 when you study the $RC$ electric circuit and, later, radioactive decay.) You will find that plotting $h(t)$ vs. time on a semilog plot (\textit{\textbf{h(t) goes on the vertical logarithmic axis, and t goes on the horizontal time axis}}) gives a straight line with a negative slope: slope = rise over run = (change in $\log_{10} h(t)$ over change in $t$). You will find it handy to plot (by hand!) your measured heights (already corrected for you for the “2.1 cm offset” and given in mm on the worksheet) and times (to be corrected by you on the worksheet to make the time when the meniscus crosses the 100 cm line be $t=0$) on the semilog graph paper that is part of the worksheet package that you must print out and take with you to the lab. Note that the pre-printed numbers on the horizontal (linear) axis should be used for the elapsed time in seconds, and the pre-printed numbers on the vertical (logarithmic) axis should be used for the $h(t)$ values in mm.

\textbf{Important: Plot your data for the orange tube and for the whitish tube on the same sheet of semilog graph paper.} This will allow you to use quick “ruler measurements” (as explained by your Lab TA) to compare slopes for the two different data sets.

If you need a YouTube tutorial on using/making plots on semilog graph paper for exponential decay, go here [http://www.youtube.com/watch?v=Tp2M9tndGG0], but if you want a clipped version of that tutorial, which removes the parts not relevant to your specific case, play the video just below here:

\textbf{Two-minute clip from YouTube video on exponential decay}

The example in the video is for radioactive decay, for which the functional form is the same as Eq. (9.8). The symbol $A_t$ in that video is your $h(t)$, and $A_0$ is your $h_0$. Furthermore, you're not counting radioactive decay events in successive time windows. You're recording the times at which the meniscus passes particular cm lines (those listed on the worksheet) on the graduated cylinder.
You should choose each flow measurement to begin at the time when the meniscus is at the 100 cm line of the graduated cylinder. A column is provided on the worksheet for dealing with this “time offset”.

Likewise, each $h(t)$ value will be the height in mm of the meniscus above the 2.1 cm line, the ingress location of the flow-tube-under-test. You must use the values for $h$ in mm on the worksheet column that takes into account the 2.1 cm subtraction from the printed white numbers on the graduated cylinder for which you measured the time when the meniscus crossed the white lines corresponding to them! Note that the worksheet already has a column that did this for you.

Using semilog plots to compare the two runs

As instructed by your Lab TA, make two plots on one sheet of semilog graph paper that is included in the worksheet package you should have printed out and brought with you to the lab room. You will immediately notice that the “orange tube” plot and “whitish tube” plot are different in the following ways: (1) the orange-tube plot drops much more quickly than the whitish-tube plot; (2) the whitish-tube plot has only one (negative) slope; (3) the orange-tube plot has three different regions: (i) an initial region with a (reasonably) constant slope that is more “gentle” (less negative) than (ii) the final region, which has a steeper (more negative) slope; (iii) a “transition” region where the slope changes from that in (i) to that in (ii).

To prepare yourself before lab for making such semilog plots and finding the decay constant $\lambda$ from them, read Sec. 4.5, Exponential Relationships, on pp. 26 and 27 at this link [http://hyperion.uregina.ca/~szymanuss/uglabs/companion/Ch4_Graph_Anal.pdf] to material from a laboratory course at the University of Regina. The material on those pages gives a table of amplitudes and times for an exponential decay, and it explains via its Fig. 4.6 (linear plot of the data) and Fig. 4.7 (semilog plot of the data) how to find the slope directly from the semilog plot and, thereby, the decay constant $\lambda$. You can use this technique in the next section to find the slope for lastO, the slope for $W$, and the slope ratio $lastO/W$.

Use the difference in slopes to determine the diameter of the whitish tube

Poiseuille’s Law applies to all of the whitish-tube plot but only to region 3(ii) of the orange-tube plot. As mentioned above, the flow in region (i) is turbulent. Displaying a wonderful example of the utility of using dye jets for seeding, and thereby visualizing, patterns in a fluid flow, the “3-clip video” shows at 20’04” that the onset on turbulence occurs near the sharp “ingress edge” of the flow tube. Figure 5 shows that both our orange tube and whitish tubes have sharp ingress edges. However, the onset of turbulence at the edge depends on the flow velocity as parametrized by the dimensionless “Reynold’s number”

$$R_e = \frac{\rho v_2 D_2^2}{\eta} = \frac{v_2 D_2^2}{\nu}$$

: low $R_e$, no turbulence; high $R_e$, turbulence. How “low” and how “high” depends on the details of the specific flow, but since $R_e$ increases with $v_2$, the average speed of the flow through the capillary-flow-tube-under-test, whichever tube – orange or whitish – has the highest flow speed at a given pressure will be more susceptible to the onset of turbulence. Equations (9.1) and (9.2) show that, all else held constant, $Q$ increases with the fourth power of $R_e$, the radius ($= D_2/2$) of the capillary tube, and $v_2$ increases with the square of $R_e$, and $R_e$ increases with the cube of $R_e$. Therefore, the tube with the larger diameter – the orange tube – is more susceptible to the onset of turbulence. Though our textbook KJF2 doesn’t mention the Reynolds Number, it should be clear from the 3-clip video that one cannot understand very much about fluid flow unless one knows about the Reynolds Number. It will help you in any future you will have in the life sciences or healthcare to know about fluid flow. Therefore, it’s worth your while now to make sure you understand the importance of the Reynolds Number for fluid flows.

Eq. (9.1) and the same line of reasoning as above explains why arterial blood flow in your body can be turbulent whereas venous blood is not.

For your final calculation, you need to divide the “last orange-tube slope”, (3)(ii) above, by the “whitish-tube slope” (2). Call this ratio $lastO/W$. You already know the diameter of the orange tube, $D_O = 1.83$ mm. Divide this by the fourth-root of $lastO/W$ and you should get a good estimate for the diameter of the whitish tube, $D_w$, which you did not know. Put that value on your worksheet.
You should be able to explain clearly to your Lab TA why the procedure described in the previous paragraph should provide a good estimate for $D_\parallel$. Your Lab TA knows the actual value of $D_\parallel$ and looks forward to seeing how close you come to it.