PHY124 Lab 10 - Nuclear gamma rays and decay

The purpose of this laboratory is the study of a random process that follows the Poisson probability distribution function \( \text{Po}(\lambda) \). The random process in this lab is gamma radioactive decay of excited nuclei (see SV8, Chap. 29.3 and 29.4).

Using the notation introduced in the videos below, the random variable will be the number of decays \( X \) (non-negative integers, including zero) occurring in a particular time period \( T \). The number \( X \) will fluctuate, but averaged over a large number of trials of the same duration \( T \), it will have a mean labeled \( \lambda \), a positive, real number. (The corresponding mean rate, i.e., average number of decays per unit time, would be \( \lambda/T \).) According to the Poisson distribution, the probability for the number of decays in time \( T \) to be the integer \( r \), given that the mean number in this period is \( \lambda \), is

\[
P(X = r) = \frac{e^{-\lambda \lambda^r}}{r!},
\]

(10.1)

where \( e \) is the base of the natural logarithm, and \( ! \) is the “factorial” symbol, e.g.,

\( 4! = 4 \times 3 \times 2 \times 1 = 24. \)

**BEFORE GOING FURTHER:** You will need an introduction to the Poisson probability distribution function \( \text{Po}(\lambda) \). **NOTE:** To avoid confusion with the symbol \( \lambda \) used later in this lab manual for the “absorption coefficient”, we use upper-case \( \Lambda \) for the mean of the Poisson distribution. The videos, mentioned next use lower-case \( \lambda \) for this mean. Please try to avoid the possible confusion. We use upper-case \( \Lambda \) for the mean.

The easiest way to do this is to watch a series of two YouTube videos called Statistics: Poisson Distribution - Introduction. In these videos the distribution is symbolized by \( \text{Po}(\lambda) \).

Watching the first one is mandatory. Lasting a little over 12 minutes (after the opening advertisement), it is at [http://www.youtube.com/watch?v=2zK3KpV3bx4](http://www.youtube.com/watch?v=2zK3KpV3bx4). (You may find the following "translations" useful: r.v. = random variable, and "naught" means zero. The speaker is British.)

Watching the second one will also be useful. Lasting a bit under 8 minutes (after the opening advertisement), it is at [http://www.youtube.com/watch?v=AIEzP_7hG14&feature=channel](http://www.youtube.com/watch?v=AIEzP_7hG14&feature=channel). (Again, r.v. = random variable. “Fortnight” means two weeks.)

If you decide after viewing these videos that you would like to have a copy of the Poisson Cumulative Distribution tables mentioned in the second video, you can find them at [http://www.unige.ch/ses/metri/statprob1/files/TablePoisson.pdf](http://www.unige.ch/ses/metri/statprob1/files/TablePoisson.pdf).

In this lab rather than using programs built in to the webpage, as was done in previous labs, you will use an Excel spreadsheet that you can either download from here [http://www.ic.sunysb.edu/Class/phy122ps/labs/124lab10updated.xls](http://www.ic.sunysb.edu/Class/phy122ps/labs/124lab10updated.xls).
IMPORTANT UPDATE: Do NOT use a spreadsheet you may find on the desktop of the lab computer. That one is obsolete. Use the one you download from the link in the previous paragraph. You will enter your data into that spreadsheet and carry out the calculations there.

There is no other worksheet that you need to print out for this lab.

If you need the .pdf version of these instructions you can get them here [http://www.ic.sunysb.edu/Class/phy122ps/labs/dokuwiki/pdfs/phy124lab10updated20101204.pdf], but they will not have active links to the web sites.

Video (introduction to this lab)

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Equipment

- one “Geiger” counter (detector and control box)
- 1 blue/yellow or orange $^{137}$Cs radioactive source ("long" half-life: $T_{1/2} = 30$ Years)
- 1 $^{137}$Ba low-intensity, liquid source in a sample vial prepared by the instructor ("short" half-life: $T_{1/2} = 2.6$ minutes)
- stopwatch
- lead absorber plates (one with 1 mm thickness, 6 with 3 mm thickness) (5 sets in A-120, to be shared)
Introduction

Though the radioactive sources in this experiment are NOT dangerous, it is important that you return them to the TA at the end of the lab. NOTE: Absolutely no food nor drink is to be allowed in the lab room. After handling radioactive sources and the lead plates, you should wash your hands before eating.

A common radiation detector (a “Geiger counter”) will be used in these experiments to count γ-rays. (Historical note: “Geiger” is the Hans Geiger who did the famous experiment with Ernest Marsden that is described briefly in SV8, Chap. 28.1. Both were supervised by Ernest Rutherford and used the particle detection technique common to nearly all the experiments carried out in Rutherford's lab, viz., “eyeball” detection of “scintillations” – flashes of light – made when energetic particles slammed into a material such as zinc sulfide. Geiger must have tired of that tedious method of particle detection and then decided to invent a better way. The “Geiger-Mueller tube” opened the door to “all-electronic” methods for detecting particles. Geiger sent one of the first ones he made in Germany around 1930 to Rutherford, and there it was soon put to good use by Cockcroft and Walton in their 1932 nuclear-transmutation experiment that won them the 1951 Nobel Prize in Physics.)

In this lab you will do measurements on
I. Fluctuations in random counting rates.
II. γ-ray attenuation in an absorbing material, and
III. the half-life of a γ-decaying excited state of the nucleus $^{137}\text{Ba}$.

A sketch of a Geiger-Mueller tube is shown below. The tube is hermetically sealed from the outside air and contains a special “counter gas”, a mixture of atomic and molecular gases that optimize the operation of the device. Particles to be detected enter the tube through the thin window. Since
particles below a certain energy will not make it through the window, there is a lower limit to the energy of particles that can be detected. The gamma rays you will detect are far above that lower-energy limit.

The principle of operation is simple but very clever. Because particles like gamma-ray photons that make it through the thin window have energies far above the ionization potential of atoms and molecules, most of their collisions with atoms or molecules in the counter gas are “ionizing events” that produce free pairs of electrons and positive ions. Due to the voltage applied across the counter, electrons are attracted to the thin wire anode (charged positively), and ions are attracted to the cylindrical cathode (charged negatively). The coaxial geometry with the thin anode wire means that the electrons are accelerated most strongly near the anode wire by the positive potential on it. (This electrostatic geometry is not treated explicitly in SV8, Chap. 15, but the figure on the right-hand side of p. 513 in SV8 is close enough. Notice how the field lines get dense near the “tip” of the charged, pointed conductor. Dense field lines signal a large electric field, and a large electric field means a large electrostatic force on any charged particle nearby.) Preferentially near the anode wire, the electrons gain enough energy to ionize other atoms and molecules, freeing up more electrons to be accelerated and take part in the “cascade”. (This is called “gas multiplication” of the ionizing signal.) The process creates a transient current of electrons moving through the resistor $R$, which generates a voltage pulse big enough to detect and record in a “ scaler” circuit that registers each count on a neon display tube. There are two slightly different Geiger counters in the labs, but their operations are similar.

Since there are only 5 sets of lead plates in the room, which need to be shared, you may have to do Part II before Part I. Your TA will organize which part you should do first.

**Part I - Fluctuations in Random Counting Rates**

Measurements of random events are always subject to fluctuations in the number actually measured. It’s incorrect to say that the fluctuations lead to “errors” because no one is doing anything “wrong”. The fluctuations are inherent in the process. Rather, it’s more correct to say that the fluctuations in random processes inherently lead to “statistical uncertainty” in the number of counts. Nuclear decay
is a random process, i.e., it is impossible to predict exactly when a given nucleus will decay. Only averages can be predicted, e.g., “On average, in this sample, 10 nuclei decay every second.” If one repeatedly measures the number of counts accumulated in a given time interval $T$ (which should be chosen to be small compared to the half-life $T_{1/2}$ of the radioactive nucleus - see SV8, Chap. 29.3), one finds that this number of counts, an integer $X$, fluctuates about an average, $\overline{X}$, which is denoted by the symbol $\Lambda$ in the Poisson Probability Distribution function $Po(\Lambda)$ introduced at the beginning of this manual. For “Poisson statistics”, as long as the mean number of counts is large compared to one (10 is large enough, and even 5 isn’t too bad), about $2/3$ of the measurements will yield a number of counts, $X$ that lie in the interval $(\overline{X} - \sqrt{\overline{X}}) \leq X \leq (\overline{X} + \sqrt{\overline{X}})$. Another way of saying this is that the “standard deviation” in a (Poisson) counting process that yields, on average, $N$ counts, is $\sqrt{N}$. (The symbol $N$ is frequently used for “number of counts”, but this choice is arbitrary.) Therefore, an estimate (based on a finite amount of data) for the measure of

the absolute statistical uncertainty in $N$ random counts is $\sqrt{N}$.

(10.2)

In Part I, you verify these assertions for the random process of nuclear decay, and in Part II and III you use for $\mp \sqrt{N}$ as the statistical uncertainty for a number of counts, $N$.

It is straightforward to obtain, next, an estimate for the relative uncertainty in the number of counts. You need only take the ratio:

relative uncertainty in $N$ counts $= \frac{\sqrt{N}}{N} = \frac{1}{\sqrt{N}}$.

(10.3)

Notice an important consequence of Eq. (10.3). As $N$ increases, the relative uncertainty decreases, but you must pay a big price to decrease it “a lot”. Everything else remaining fixed, the number of counts recorded is proportional to the time $T$ over which one records them. If you want to decrease the relative uncertainty by a factor of 10, you need 100 times as many counts, and to obtain them, you must count for 100 times as long. If you have some way of increasing the count rate, say by getting a much more powerful radioactive source (which can be dangerous to the health of humans or other living things nearby unless a lot of “shielding” is used to protect them), or by building a bigger detector that “collects” a greater fraction of the radioactive decays, that can help to decrease the amount of counting time needed to achieve a certain, desired relative uncertainty.

In this part you use a long-lived radioactive source ($^{137}$Cs, $T_{1/2} = 30$ years) to study fluctuations in random counting rates. For this part of the lab you measure and record continuously for 10 minutes the number of counts obtained in consecutive ten-second intervals. Since the half-life of the source, 30 years, is large compared to the duration of your experiment, the average counting rate remains constant during the experiment as long as you do not move the detector or the source, do not change any of the settings on the control box, and do not bring any other radioactive source close-by, during your measurements.

Turn on the voltage in the Geiger counter control box. The HIGH VOLTAGE should be set at $\sim 600$ V. Do NOT change this setting during your experiment! It will distort your data if you do.
Notice that even when your radioactive source is far from the counter, the device still registers counts. These are “background counts”, and you must take them into account. Count the background for 5 minutes with your radioactive source far removed from the counter and write down the number of background counts you record. You will need this background-count number later, in Part III.

In this part you ignore the background because both the background (as long as it is caused by a random process that doesn't vary) and your source (as long as you don't move it) should provide (on average!) a constant counting rate. If the background counting rate is constant, it will have, on average, the same effect on each 10 second counting interval. Likely causes of the background are cosmic rays and natural radioactivity in materials in the room, even in you! (NOTE: If you have recently had a medical treatment that introduced “radioactive tracers” into your body, then “you” will increase the background, and if you move around, this will change the background counting rate for your measurements. This has actually happened in the past in one of our advanced laboratory courses.)

Position the ¹³⁷Cs source such that you get roughly 60 counts/min. You will likely have to place the source on the lab table about 15 cm away from the outside of the counting box in order to get this rate. Make sure you leave the source in this position during Part I of this experiment or else you will change the average count rate and collect “useless” data.

Now, you and a partner should observe and record the number of counts displayed over a period of 10 minutes (600 seconds). Use the continuous or manual mode on the control box for this measurement. During this 10-minute period, one of you should be watching the counter on the control box continuously and calling out to your partner at each 10 second interval the number of counts on the counter. Your partner should be recording each of these numbers. (They can be put directly into the spreadsheet.) At the end of the 10 minutes, you should have obtained 60 data points. Make sure you are taking these data continuously (No Stopping!) over the entire 10 minute time interval.

After you have collected your data, if you have not done so already enter them in the Excel spreadsheet. You should notice that the count number recorded at the end of each 10 second interval represents the total count up to that point in time. What you need for analysis of your data is not these numbers; you need the number of counts in each 10 second interval. In the notation of the two introductory videos from the internet, these may be called \( X(T = 10 \text{ s}) \). The computer will calculate these for you.

The spreadsheet now calculates the frequency of occurrence of each number of counts-per-10 seconds and plots it as a histogram. (This is similar to the way we display your grades; for each score, (scores are on the horizontal axis), the histogram plots the frequency of occurrence on the vertical axis).

The spreadsheet also calculates the average count \( \bar{X} \) and plots the Poisson distribution based on this average. You should satisfy yourself that the Poisson distribution is a good fit to the data. The sheet also calculates \( \sqrt{\bar{X}} \) and shows lines on the graph which mark the interval \( (\bar{X} - \sqrt{\bar{X}}) \leq X \leq (\bar{X} + \sqrt{\bar{X}}) \).

Is it true for your data set that about 2/3 of the number of counts-per-ten-second measurements lie between these lines?

**Part II Absorption of γ-rays**

In part II you verify that the number of γ-rays passing through an absorbing material decreases exponentially with the thickness of that material. (This was discovered in absorption of visible light and is called the “Beer-Lambert-Bourguet Law”; see click here [http://en.wikipedia.org/wiki/Beer%E2%80%93Lambert_law].)
Energetic charged particles passing through matter lose energy continuously by (quantum-) mechanical collision events. Gamma rays passing through matter lose energy by some (quantal) processes (e.g., Compton scattering, see SV8, Chap. 27.5), or they can disappear entirely via other (quantal) processes (e.g., the photoelectric effect, see SV8, Chap. 27.2, or production of particle/anti-particle “pairs” such as an electron and positron, see SV8, Chap. 30.4, for gamma-ray energies above 1.02 MeV). Roughly speaking, gamma rays either pass through the material, or they are absorbed. The number $\Delta N$ of $\gamma$-rays that are absorbed, i.e., the decrease in the number of $\gamma$-rays that survive a traversal of a thin layer $\Delta x$ of material, is proportional to the number $N$ of $\gamma$-rays incident on the material and to the thickness $\Delta x$ of the layer. One writes $\Delta N = -\lambda N \Delta x$ where $\lambda$ (NOT to be confused with the mean value $\Lambda$ in the Poisson distribution, $Po(\Lambda)$ discussed above) is a proportionality constant called the “absorption coefficient” that depends on the material density, composition, and the energy of the $\gamma$-ray. This equation looks exactly like the equation describing radioactive decay (see SV8, Chap. 29.3) if one simply replaces $\Delta x$ and $x$ with $\Delta t$ and $t$. There $\lambda$ is the decay constant. Thus, the number of $\gamma$-rays present after traversing a thickness $x$ of material is

$$N = N_0 e^{-\lambda x},$$

(10.4)

where is $N_0$ is the number of $\gamma$-rays incident. In this part you use the long-lived radioactive source ($^{137}\text{Cs}$, $T_{1/2} = 30$ years) to study absorption of $\gamma$-rays. There are 5 sets of lead absorption plates available for this part. Each set includes a 1 mm plate and 6 plates 3 mm thick. Since there are not enough sets of plates for every group, they will have to be shared; you may have to collect data for this part of the experiment before you do Part I.

Place the source holder in the bottom slot of the counting box and then place just above it a 1 mm thick lead plate. The source emits both $\gamma$ rays and $\beta$ rays (electrons emitted by the nucleus), and you want to eliminate the “background” $\beta$ rays for this part so you can study the absorption of $\gamma$ rays. The $\beta$ rays are stopped by the 1 mm plate and most of the $\gamma$ rays pass right through it. Think of this 1 mm lead plate as a “$\beta$ ray “filter”.

**Define the measurement with the 1 mm thickness lead as absorber thickness “0”. Do NOT count this 1 mm $\beta$ ray “filter” plate when you calculate the thickness of the subsequently added absorber plates.**

Add one lead plate over the source+1-mm-filter plate. For a 1 minute counting time, record the number of counts. Continue to add lead plates one at a time until you have used all the plates. Record counts for 1 minute each time you add an additional plate and enter the data on the area of the Excel spreadsheet that corresponds to Part II of this experiment.

The spreadsheet will calculate the error in each measurement ($\sqrt{N}$) and plot counts-per-one-minute interval vs. absorber thickness in mm. It will also fit these data to an exponential function, $N = N_0 e^{-\lambda x}$, viz., Eq. (10.4), allowing you to determine the absorption coefficient $\lambda$ from your data.

Does the fitted line pass within most of the error bars for your data?

When you have determined your absorption coefficient, write it on the board at the front of the lab so that everyone will be able to compare all the different values obtained by the different groups of lab partners doing the “same” experiment.

**Part III - The half-life of Barium 137**
In part III, you measure the half-life, $T_{1/2}$, of an excited state of the $^{137}$Ba nucleus. The half-life is of the order of minutes, a convenient time for this experiment. Your instructor will prepare the source for you when you are ready for this part of the experiment. Don't waste time in using it because the longer you wait, the weaker it gets. See Eq. (10.3) and the comments below it!

Set up your Geiger counter so you can begin counting as soon as your TA gives you the source. Count for 10 time intervals of 30 seconds each. As in Part I, these time intervals should be continuous, so you should count for 5 minutes, recording the number of counts after each 30 second interval. Enter your cumulative counts into the Excel spreadsheet for Part III; it will then calculate the counts per 30 second time interval, given by the symbol $N$.

In Part I you collected background counts for an interval of 5 minutes. In this part of the experiment you will be observing time intervals of 30 seconds. Using the background counts you collected for 5 minutes, determine the corresponding count rate (counts over time), and from that rate calculate the number of background counts expected for a 30 second interval and enter this in the box on the Excel spreadsheet. This spreadsheet will subtract this from your recorded data, will plot the results vs. time, and will fit these result to the $N = N_0 e^{-\lambda t}$ radioactive decay law.

Does the fitted line pass within most of the error bars?

You can get the decay constant $\lambda$ from the fit to $N = N_0 e^{-\lambda t}$ much as you did in Part II for the absorption coefficient.

The half-life $T_{1/2}$ from $N = N_0 e^{-\lambda t}$ is $T_{1/2} = \frac{\ln(1/2)}{-\lambda} = \frac{\ln 2}{\lambda}$. (See SV8, Chap. 29.3.)

Calculate the half-life and write it on the board at the front of the lab so that you will be able to compare all the different values that are obtained by different groups of lab partners.