MAP 103 First-Day Worksheet

In this first-day worksheet, my aim is to integrate some review topics with some material that you will be held responsible for in this course. Below are some of the basic assumptions about the real numbers and equations involving real-number quantities.

(1) Ordering Property. The real numbers are ordered. Another way to say this is that, given two numbers a and b, one and only one of the following is true:

- $a > b$
- $a < b$
- $a = b$

(2) The Real Number Line. A real number can be thought of as a point on a line. Again, this reinforces the fact that the real numbers are ordered.

(3) Addition. Two real numbers can be added together. One can think of adding as a form of counting in groups.

(4) Commutative Property of Addition. The commutative property of addition (in words) states that order doesn't matter when adding two numbers together. The mathematical statement is the following:

- For any two real numbers a and b, $a + b = b + a$.

(5) Associative Property of Addition. This property talks about the fact that it doesn't matter what order you complete two consecutive addition problems. For this property, the mathematical statement is actually easier to understand:

- For any three real numbers a, b, and c, $(a + b) + c = a + (b + c)$.

Question: What do the parenthesis tell us to do?

(6) Multiplication. Two real numbers can be multiplied together. One can think of multiplying as a form of doing a large number of consecutive addition problems.

(7) Commutative Property of Multiplication. The commutative property of multiplication (in words) states that order doesn't matter when multiplying two numbers together. The mathematical statement is the following:

- For any two real numbers a and b, $a \cdot b = b \cdot a$.

Question: Do we need the multiplication dots?

(8) Associative Property of Multiplication. This property talks about the fact that it doesn't matter what order you complete two consecutive multiplication problems. For this property, the mathematical statement is actually easier to understand:

- For any three real numbers a, b, and c, $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.

(9) Distributive Property. This property connects the operations of addition and multiplication, and is essential to almost all of the "difficult" algebra we will see in this course. In words, it says that the product of a sum is the sum of the products. The mathematical statement appears below:

- For any three real numbers a, b, and c, $a(b + c) = ab + ac$.

Questions: What is a product? What is a sum?

(10) Identity Property of 0. The number 0 is said to be the additive identity because we can add 0 to any real number and not change the original number's quantity. Mathematically:

- For any real number a, $a + 0 = 0 + a = a$.

(11) Identity Property of 1. The number 1 is said to be the multiplicative identity because we can multiply by 1 to any real number and not change the original number's quantity. Mathematically:

- For any real number a, $a(1) = (1)a = a$.

(12) Additive Inverse Property. There is a number, denoted $-a$ and called the opposite of a, to which we can add a to get 0. Mathematically:

- For any real number a, there exists a number $-a$ so that $a + -a = -a + a = 0$.

(13) Multiplicative Inverse Property. There is a number, denoted $\frac{1}{a}$ and called the reciprocal of a, by which we can multiply a to get 1, so long as $a \neq 0$. Mathematically:

- For any real number $a \neq 0$, there exists a number $\frac{1}{a}$ so that $a\frac{1}{a} = \frac{1}{a}a = 1$.

(14) Distributive Property. This property connects the operations of addition and multiplication, and is essential to almost all of the "difficult" algebra we will see in this course. In words, it says that the product of a sum is the sum of the products. The mathematical statement appears below:

There is also this notion of equality, and there are two properties of equality that are absolutely essential to understand.

(15) Addition Property of Equality. In words, this says that if two amounts are equal, adding (or subtracting) equal amounts from them results in equal amounts. The mathematical statement:

- For any three real numbers a, b, and c, $a = b$ if and only if $a + c = b + c$.

(16) Multiplication Property of Equality. In words, this says that if two amounts are equal, multiplying (or dividing by) equal amounts from them results in equal amounts, except when multiplying or dividing by 0. The mathematical statement:

- For any three real numbers a, b, and c and $c \neq 0$, $a = b$ if and only if $ac = bc$.

Question: Why can't we divide by 0?

Now that you have the properties, try and solve the following problems with a partner or two:

(1) For properties (4), (5), (7), (8), (9), (10), (11), (12), (13), (14), and (15) make up two statements involving numbers that represent the statement's truth.

(2) I enjoy solving equations, but I don't know why I do what I do. Look at the solution to solve the equation $2(x + 3 + x) = x + 9$ and give some justification for each step:

$$2(x + 3 + x) = x + 9 \quad (1)$$
$$2(x + x + 3) = x + 9 \quad (2)$$
$$2(2x + 3) = x + 9 \quad (3)$$
$$2 \cdot 2x + 2 \cdot 3 = x + 9 \quad (4)$$
$$4x + 6 = x + 9 \quad (5)$$
$$4x - x + 6 = x - x + 9 \quad (6)$$
$$3x + 6 = 9 \quad (7)$$
$$3x + 6 - 6 = 9 - 6 \quad (8)$$
$$3x = 3 \quad (9)$$
$$x = 1 \quad (10)$$