

MAP 103 First-Day Worksheet Answers to Problem 2

- (1) For properties (4), (5), (7), (8), (9), (10), (11), (12), (13), (14), and (15) make up two statements involving numbers that represent the statement's truth.
- (2) I enjoy solving equations, but I don't know why I do what I do. Look at the solution to solve the equation $2(x + 3 + x) = x + 9$ and give some justification for each step:

$$2(x + 3 + x) = x + 9 \quad (1)$$

$$2(x + x + 3) = x + 9 \quad (2)$$

$$2(2x + 3) = x + 9 \quad (3)$$

$$2 \cdot 2x + 2 \cdot 3 = x + 9 \quad (4)$$

$$4x + 6 = x + 9 \quad (5)$$

$$4x - x + 6 = x - x + 9 \quad (6)$$

$$3x + 6 = 9 \quad (7)$$

$$3x + 6 - 6 = 9 - 6 \quad (8)$$

$$3x = 3 \quad (9)$$

$$x = 1 \quad (10)$$

Step 1 to Step 2 What happened? We switched x and 3. We replaced $3 + x$ with $x + 3$, since $3 + x = x + 3$, and we used the commutative property of addition.

Step 2 to Step 3 $x + x$ suddenly became $2x$. How did this happen? Many of you mentioned "combining like terms", but that's not a property of real numbers. Let's see. $x + x = x \cdot 1 + x \cdot 1$ by the identity property of 1. Now, we can "factor out" x from both terms. This works because of the distributive property, since $x(1) + x(1) = x(1 + 1) = x \cdot 2 = 2x$ (the last manipulation is by the commutative property of multiplication).

Step 3 to Step 4 We have distributed the 2 through the parenthesis. This is the distributive property.

Step 4 to Step 5 We really haven't used any properties in particular here except just to multiply together two numbers.

Step 5 to Step 6 What happened? x was subtracted from both sides. The equality property of addition allows us to do this since addition and subtraction are really the same thing. Why did I choose to subtract x from both sides? It will eliminate x on the right-hand side (RHS). $-x$ is the quantity that should be added to x to get 0. This is the additive inverse property.

Step 6 to Step 7 Here we have performed the arithmetic $x - x = 0$ and then added $0 + 9 = 9$. So we have used the identity property of 0 to get 9 alone on the RHS. Look at step 2 to step 3 and try to figure out how we get that $4x - x = 3x$.

Step 7 to Step 8 Similar to the reasoning for step 5 to step 6. We can subtract 6 from both sides by the additive property of equality. Then subtracting 6 will eliminate 6 from the left-hand side (LHS).

Step 8 to Step 9 Arithmetic is performed on the RHS to get 3. $6 - 6 = 0$ by the additive inverse property.

Step 9 to Step 10 We've divided both sides by 3 (or multiplied both sides by $\frac{1}{3}$) and that is allowed by the multiplication property of equality. Then, $3 \cdot \frac{1}{3} = 1$ by the multiplicative inverse property. $1 \cdot x = x$ by the identity property of 1 and the RHS is simplified by arithmetic.