The purpose of this lab is to measure the period of a simple pendulum and to begin to learn how to find, investigate, and understand major sources of uncertainties and error in a protocol of measurement and data analysis.

A simple pendulum consists of a mass \( m \) (ideally, a point) having gravitational weight (a vector) \( \mathbf{w} = mg \), suspended from a fixed point by a string (ideally, massless) of length \( L \). (For your convenience the figure also shows the two components of \( \mathbf{w} \), one parallel to the string and one perpendicular to the string.) If set into motion with care, the mass swings along an arc (labeled by \( S \)) lying in a plane that contains the gravity vector \( \mathbf{g} \). \( \theta(t) \) is the time-dependent angle that the taut string makes relative to the vertical defined by the direction \( \mathbf{g} \). If it’s NOT set carefully into motion, it becomes a conical pendulum, which does not swing in a plane and is, therefore, a more complicated physical system. You want to make sure your simple pendulum swings in a plane.

The period of this motion is defined as the time \( T \) needed for the mass to swing back and forth once. We will see later in this course that the approximate relation between the period \( T \) and length \( L \) of the pendulum is

\[
T = 2\pi \sqrt{\frac{L}{g}},
\]

where the magnitude of \( g \) is 9.81 m/s\(^2\). In the derivation of this equation the assumption is made that the angle \( \theta \) is small so that \( \sin \theta \approx \theta \) (where \( \theta \) is measured in radians).

By measuring the period \( T \) of oscillation of the pendulum as a function of \( L \), the length in the string, you can determine a value for \( g \), the acceleration due to gravity. The different quantities that are important in the experiment: \( L, \theta, \) and \( T \). Note that to try to minimize random errors, you should measure \( L \) several times and the time it takes for 10 oscillations rather than just one oscillation.

If the computer on your bench is working and has the program “Shortcut to SnapTimePro” installed, use it as your timer. If not, use a stopwatch.
Equipment

- Pendulum: 1 steel ball, 1 holder, 1 string
- Protractor (to measure angles)
- Computer or stopwatch: to be used as a timer
- Ruler: (to measure length)

Estimating the main errors in the experiment

Error in the length of the string

You should make an estimate of the error in you measurement of the length string. Factors to take into account should include the scale on the ruler, but also how accurately you can determine the center of the ball and the pivot point.

An approach you can take is to first measure the length from the center of the ball to the center of the pivot directly. Then to obtain a second measurement measure from the top of the pivot to the bottom of the ball, and then from the bottom of the pivot to the top of the ball, and take the average of these two values. The difference between these measurements is a reasonable estimate for the error in the length of the string. (To save time you can do this once, and the use the error value for all your subsequent measurements of the length which you need only measure once directly).

Error in the period

If we measure the time for 10 oscillations we can find the time for one oscillation simply by dividing by 10. Now we need to make an estimate of the error.

First you need to estimate the error in your measurement. How accurately do you think you can press the button to tell the computer when to start and stop the measurement? Let’s say that you think you can press the button within 0.2 seconds of either the start or the stop of the measurement. You need to account for the errors both times, but as we discussed earlier, because these errors are random they add in quadrature so you can say that

\[
\sigma_{10T} = \sqrt{0.2^2 + 0.2^2} = 0.28 \text{ s}
\]

Now we find the error in T by dividing by 10

\[
\sigma_T = \frac{\sigma_{10T}}{10} = \frac{0.28}{10} = 0.028 \text{ s}
\]

So you can see it was a good idea to measure several periods instead of one, we get a much more accurate result. Maybe you'd like to think about why we don't measure 100 oscillations (and because you'd get bored is only part of the answer!).

How accurately do you think you can press the button, is 0.2 seconds an overestimate or underestimate for your reaction time? If you think that the accuracy of your button press is different to 0.2 seconds you should work out what you think \(\sigma_T\) is if you make a measurement of 10 oscillations.

The effect of angle on the period of oscillation

We mentioned above that the equation we want to test is only valid for small angles. The first measurement
we will make is to measure the period of oscillation for 3 angles of release.

For a single length of the string choose three angles at \(15^\circ\), \(30^\circ\) and \(80^\circ\) and measure the time for 10 periods for each angle.

Taking into account your estimate for the error in \(T\) above are the periods the same for all 3 angles?

**Taking data**

Now we have some idea about the errors involved in our experiment we should take some data. You will need to measure the period of oscillation of the pendulum for different values of the string length \(L\). You will want to have the same angle \(\theta\) for each \(L\). So the procedure you should follow is to set your string length to a given \(L\), which you should write in the table on your worksheet, and then pull it back to angle of \(15^\circ\) which you can measure with the protractor. After releasing the pendulum you should measure, using the computer stopwatch, the time for 10 oscillations and record that on the worksheet as well. Do this for 10 different lengths of the string \(L\). Once you have your data for \(10T\) you can then work out what \(T\) is for each \(L\) simply by dividing by 10. You should also enter in to your table values for \(\sigma_L\) and \(\sigma_T\) based on the estimates you made earlier.

**Making a plot of our data**

Use the Plotting Tool to make these plots. There are two computers with web access in the lab and many more in A129 (the login for these computers is written on the monitor and the password is “mastering”). Or you can use your own computer.

**Linear Plot**

Recall that we said earlier that we expect that \(T = 2\pi \sqrt{\frac{L}{g}}\). We can rearrange this as \(L = \frac{g}{(2\pi)^2} T^2\),

which means that we should get a straight line if we plot \(L\) against \(T^2\) and extract from it the value of the acceleration due to gravity.

To test this we need to work out what \(T^2\) is for each value of \(L\) and of course we need to know what the error in \(T^2\), \(\sigma_{T^2}\), is so that we can draw error bars on the graph. So you need to complete the last to columns of your table. Finding \(T^2\) is easy enough. To obtain \(\sigma_{T^2}\) we will need to propagate the error in \(T\), \(\sigma_T\) using some of the equations above. Equation (1.8) tells us how to propagate the error of a quantity in to the error of the power of that quantity.

In this case the relative error in \(T^2\), which is \(\frac{\sigma_{T^2}}{T^2}\) is the same as twice the relative error in \(T\), which is \(\frac{\sigma_T}{T}\).

You can thus find that \(\sigma_{T^2} = 2T\sigma_T\)

Make sure you can understand how to get this equation from the equations in your error manual!

**Log–Log Plot**

In the plot above we used our knowledge of the expected relation ship to deduce we should plot \(L\) against \(T^2\). But what if we had not known this and instead wanted to find the power–law expression to fit our data. Or perhaps we just want to check that \(T^2\) actually does represent the best fit to the data. A good way of testing for a particular power law dependency of one quantity on another is by making a log–log plot.
If we expect \( y \) to depend on \( x \) via a general relationship

\[ y = Ax^n \]

and we take logarithms of both sides, we find that:

\[ \ln y = \ln(Ax^n) = \ln(A) + \ln(x^n) = \ln(A) + n \ln(x) \]

So if we plot the logarithm of \( y \) against the logarithm of \( y \) we should be able to extract the exponent that best fits our data from the slope of the graph and the prefactor from the intercept.

To work out the error in a logarithm of a quantity from the error in that quantity we need to use the formula we derived using calculus in Lecture 1.

\[ \sigma_f = \left| \frac{df}{dx} \right| \sigma_x \]

In the case of \( f(x) = \ln(x) \), \( \frac{df}{dx} = \frac{1}{x} \), so that the error in \( \ln(x) \) which is \( \sigma_{\ln(x)} = \frac{\sigma_x}{x} \).

Convert your values to their natural logarithms, find the error in these quantities, and plot \( \ln(L) \) against \( \ln(T) \). Does an exponent of 2 fall within the experimentally measured exponent as given by your graph?

If you convert the intercept of the graph back to \( A \), which is \( \frac{g}{4\pi^2} \), you can again obtain the value for \( g \) and compare it to the value you got from your graph against \( T^2 \). (Don't forget to convert back your error in \( \ln(A) \) to the error in \( A \) by reversing the error relationship you used before, ie. \( \sigma_x = x\sigma_{\ln(x)} \).) Which plot gives a better value for \( g \)? Discuss!

**Number crunching tool**

You can use this tool to convert your \( l \) and \( t \) values and their errors in to the things you need to plot. Don't forget to appropriately round your values, just like you would if you were using your calculator! (Note if you leave rows blank you will get error messages, but don't be worried about these, they don't affect the calculations for the other rows.)

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