The purpose of this lab is to study simple harmonic motion of a system consisting of a mass attached to a spring. You will establish the relationship between period, mass, and spring constant.

Equipment

- air track with springs and glider
- interface box
- photo gate
- weights
- 2 springs
- computer
- scale to weigh the glider
Introduction

In this lab the phenomenon of Simple Harmonic Motion will be studied for masses on springs. The physical basis of these oscillations is that the force exerted on the mass by the spring is proportional to and in the opposite direction to the displacement of the mass from equilibrium.

Part I Measurement of the Spring Constant

In this lab, you measure the spring constant $k$ of two springs attached a glider on an airtrack and attached to the end of the track. You measure the spring constant $k$ in Hooke’s law for the two springs combined. (The reason for mounting two springs is that these two springs are used in Part II and III of this lab.)

$$F = -kx$$  \hspace{1cm} (7.1)
The force $F$ is supplied by the weight suspended from the string (see Fig 2. above).

$$F = mg$$  \hspace{1cm} (7.2)

You suspend various weights from the string and measure the displacement of the glider from its equilibrium position define as the position when a 100 gram weight is suspended initially.

Before you start ensure your air track is level. Attach a piece of string to the glider and lay it on the pulley with a mass $m$ of 100g suspended from the free end of the string as shown above. Record the position of your glider for the 100g weight and define this as your equilibrium position. Be sure that the string moves freely and does not scrape. Repeat the measurement with additional mass, adding 20g 4 times for a total of 4 measurements of displacement from equilibrium. Make sure you are taking the difference between your position readings and the equilibrium position as your data. Enter the values for the additional mass $\delta m$, the additional displacement $\delta x$ and the additional force $\delta F$ in your notebook. Calculate for each of the 4 sets of numbers the slopes $k = \frac{\delta F}{\delta x}$. Calculate the average $k$ and its absolute error $\sigma_k$ (use $(\text{max} - \text{min})/2$ for the error).

**Part II Measurement of the Period of Glider/Spring System**

In this part, you measure the period $T$ of an oscillation caused by the two springs from Part I. The period of a mass–spring system is given by

$$T = 2\pi\sqrt{\frac{m}{k}}$$  \hspace{1cm} (7.3)

You should note when we have both springs attached as we do in our experiment the $k$ in this equation is the sum of the spring constants of each spring. This sum is what we measured in the first part of the lab.

Remove the string from the glider and measure the mass $M$ of the glider with the scale and record it on your notebook (neglect error in the mass $M$).

Attach the glider to the two springs and place the glider on the air track so that the ‘metal flag’ atop the glider is centered at the equilibrium position.

On the computer, go to “MISCELLANEOUS TIMING MODES” on the Main Menu screen, then chose “PENDULUM TIMER”, then “Normal Time Display”. This mode allows the computer to measure the period $T$. Make sure the “SET” button is pushed on the gray interface box. Set the glider in motion by displacing it from the equilibrium position slightly and make sure that for all oscillations, the flag on top of the glider completely passes through the photo gate. Push the “STOP” button on the gray interface box when you see ~10 rows of data on the computer screen. Then hit “ENTER” and select “Display Table of Data”. Scroll down to the ‘Mean’ value and record it in your notebook.

Repeat the above three times, each time adding an additional 20 grams on top of the glider. Record the values of the total mass (glider and additional mass combined) and the periods in your notebook.

Calculate the theoretical period $T_{\text{theor}}$ using the equation (7.3).

The error in $T_{\text{theor}}$ can be calculated from the the error in $k$ you estimate in Part I. From equation 7.3 and equation (E.8) of the Error Manual, if we take into account only the error of the spring constant $\sigma_k$ you can find that

$$\sigma_{T_{\text{theor}}} = T_{\text{theor}} \frac{1}{2} \frac{\sigma_k}{k}.$$  

Use the plotting tool to plot $T_{\text{theor}}$ vs $T_{\text{meas}}$, including error bars for $T_{\text{theor}}$. Record the slope and it's error in your notebook.

**Part III Potential Energy in the Spring**

In this part of the lab, you observe energy conservation. You will see that the maximum potential energy in the spring at maximum displacement from equilibrium
equals the maximum kinetic energy when the glider goes through the equilibrium point.

\[ KE_{max} = \frac{1}{2} m v_0^2 \]  \hspace{1cm} (7.5)

The setup is the same as in Part II,

On the PC screen select “Gate Timing Modes”, “One Gate”, “Normal Time Display”. Push the “SET” button on the grey interface box. Displace the glider by \( \sim 30 \text{ cm} \) from equilibrium and enter the glider displacement in your notebook. Use a 3 mm error for it. Release the glider and record a few time values on the PC screen. They should be only increasing very slowly if the track is frictionless. These are the times during which the flag on the glider blocks the photo gate beam and are the transition times. Record the first time (or the second if the first is very different to the other times) in your notebook. Calculate the maximum velocity \( v = \frac{w}{t} \) of the glider from the width \( w \) of the glider flag and the recorded transition time \( t \). We can neglect the error in the time, meaning that the relative error in the velocity is the same as the relative error in the flag width, i.e. \( \sigma_v = v \frac{\sigma_w}{w} \).

Now we need to check if energy is conserved.

- Calculate the maximum potential energy originally stored in the spring using equation (7.4) and record it in your notebook. Also calculate the error \( \sigma_{PE_{max}} = 2 PE_{max} \frac{\sigma_{x_0}}{x_0} \).
- Calculate the kinetic energy the object has when it is moving with its maximum velocity \( v_0 \) using equation (7.5) and record it in your notebook. Also calculate the error \( \sigma_{KE_{max}} = 2 KE_{max} \frac{\sigma_{v_0}}{v_0} \).

Are the maximum potential energy and kinetic energy the same within error? What does this mean for conservation of mechanical energy in simple harmonic motion?