Total Energy Conservation

Non conservative forces remove mechanical energy from the system, but it is not destroyed, it is simply converted to a different form of energy (frequently, but not always, heat).

The total energy conservation law can also be useful, for example when a frictional force $F_{fr}$ is acting and an object travels a distance $d$ while it goes from a height $h_1$ to $h_2$, changing its velocity from $v_1$ to $v_2$, conservation of total energy tells us

$$\frac{1}{2} m v_1^2 + m g h_1 = \frac{1}{2} m v_2^2 + m g h_2 + F_{fr}d$$

Snow Bike

Snow Bike Video

If the cyclist starts from rest then what is his velocity at the bottom of the hill?

How far will he travel down the road before coming to a stop?

Gravitational Potential Energy over Long Distances

As we saw previously the force due to gravity on an object of mass $m$ due to an object of mass $M$ at distance $r$ is

$$-\frac{GmM}{r^2} \hat{r}$$

If we raise an object from the surface of a planet to a distance $h$ above it the change in potential energy is

$$\Delta U = \int_{R_E}^{R_E+h} \frac{GmM}{r^2} dr = -\frac{GmM}{R_E+h} + \frac{GmM}{R_E}$$
We should consider where a suitable zero of potential energy is

Choosing \( r = \infty \) as our reference position is attractive as then the potential energy change in bringing an object infinity to a position \( r \)

\[
\Delta U = \int_{\infty}^{R} \frac{G m M}{r^2} \, dr = -\frac{G m M}{r}
\]

is the same as the potential energy, so generally we say the gravitational potential energy at point \( r \) from the center of a mass \( M \).

\[
U = -\frac{G m M}{r}
\]

**Escape velocity using force**

\[
\frac{dv}{dt} = \frac{-G M_E}{(R_E + x)^2}
\]

\[
\frac{dv}{dx} \frac{dx}{dt} = v \frac{dv}{dx} = \frac{-G M_E}{(R_E + x)^2}
\]

\[
\int_{v_0}^{v} v \, dv = \int_{0}^{x} \frac{-G M_E}{(R_E + x)^2} \, dx
\]

\[
\frac{1}{2} (v^2 - v_0^2) = \frac{G M_E}{R_E + x} - \frac{G M_E}{R_E}
\]

\[
v^2 = v_0^2 + 2 \left( \frac{G M_E}{R_E + x} - \frac{G M_E}{R_E} \right)
\]

as \( x \rightarrow \infty \) the object will escape if

\[
0 \geq v_0^2 - \frac{2 G M_E}{R_E}
\]

For the object to escape

\[
v_0 \geq \sqrt{\frac{2 G M_E}{R_E}}
\]

**Escape velocity using potential energy**

At the Earth's surface the total mechanical energy of a rocket is

\[
\frac{1}{2} m v^2 - \frac{G m M_E}{R_E}
\]

The condition for escape is that infinity the velocity equals zero (just avoids being negative). As at infinity the gravitational potential energy is equal to zero the condition for the escape velocity is

\[
\frac{1}{2} m v_{\text{escape}}^2 - \frac{G m M_E}{R_E} = 0
\]

and

\[
v_{\text{escape}} = \sqrt{\frac{2 G M_E}{R_E}}
\]
Space Elevator

The escape velocity is very large and launching a rocket is not a very sophisticated solution. (And it doesn’t always work)

A Space Elevator is a possible alternative.

A space elevator would not reduce the amount of work needed to be done to get something in to space. The appeal of a space elevator is the ability to control the rate at which work is done.

Power

Power is defined as the rate at which work is done.

Average power is given by

\[ P = \frac{W}{t} \]

Instantaneous power is given by

\[ P = \frac{dW}{dt} \]

Recall that \( dW = \vec{F} \cdot d\vec{l} \), which means that

\[ P = \frac{dW}{dt} = \vec{F} \cdot \frac{d\vec{l}}{dt} = \vec{F} \cdot \vec{v} \]

Units of Power are J/s or W (Watts).

Power to drive up a slope

How much power is required from the engine to drive a car up a plane (with friction) at a constant velocity?

We can assume the the applied force from the engine is directed up the plane. It needs to be equal and opposite to the forces down the plane.

\[ F_A = mg \sin \theta + \mu mg \cos \theta \]

\[ P = (mg \sin \theta + \mu mg \cos \theta)v \]
The amount of power required to do a certain amount of work in a certain amount of time is not adjustable. However, a bicycle allows you to adjust the manner in which you produce the power. Our legs are not very good at moving at very high speed, because it is difficult for us to move them more than a certain number of times per minute and the stride length is obviously limited.

Suppose the net force opposing your motion is $F_{\text{opp}}$. To move at a velocity $v$ you need to produce a velocity an amount of power $F_{\text{opp}}v$.

\[
F_P v_P = F_{\text{opp}} v_W \\
F_P = F_{\text{opp}} \frac{v_W}{v_P} = F_{\text{opp}} \frac{r_W}{r_{FW}} \frac{r_{CW}}{r_P} v_P
\]

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**Cadence**

Is it always best to push big gears?

If you can train yourself to run at a high cadence you can achieve the same power with lower force per pedal push when cycling in a lower gear.

*Lance Armstrong and Jan Ullrich*