| Review | Functions | Examples |
C. Review of Calculus (for this course)

- **tangent** = instantaneous slope
- \( \frac{\Delta f}{\Delta t} = \) average slope = rate of change of \( f(t) \) with \( t \)

**variable function**: rule for assigning to each value of \( t \) a value \( f(t) \)
C. Review of Calculus

Derivative of $f(t)$: \[ \frac{df(t)}{dt} \]

= slope of the tangent on $f(t)$

(C.1) \[ \frac{df(t)}{dt} \equiv \lim_{\Delta t \to 0} \frac{\Delta f}{\Delta t} \] is a function of $t$

\[ \frac{df(t)}{dt} \] gives the rate of change of $f(t)$
C. Review of Calculus

Rules for taking the derivatives

constant factor goes in front of the derivative

\[
\frac{d(a \cdot f(t))}{dt} = a \cdot \frac{df(t)}{dt}
\]

derivative of sum = sum of derivatives

\[
\frac{d(f(t) + g(t))}{dt} = \frac{df(t)}{dt} + \frac{dg(t)}{dt}
\]
C. Review of Calculus

Chain Rule: multiply the derivatives of inner and outer functions

\[
\frac{d(f(g(t)))}{dt} = \frac{df(g)}{dg} \cdot \frac{dg(t)}{dt}
\]

(C.3)
C. Review of Calculus

functions we use:

\( a = \text{constant} \)
\( t = \text{variable} \)

(C.4) \[ \frac{da}{dt} = 0 \]

(C.5) \[ \frac{d(t^n)}{dt} = n \cdot t^{n-1} \quad (n = \text{integer}) \]

(C.6) \[ \frac{d(\sin t)}{dt} = \cos t \]
C. Review of Calculus

\[(C.7) \quad \frac{d(\cos t)}{dt} = -\sin t\]

\[(C.8) \quad \frac{d(e^t)}{dt} = e^t\]
C. Review of Calculus

Examples: $a, b$ are constants

example (e.1): $f(t) = a + b \cdot t^2$

\[
\frac{df(t)}{dt} = \frac{d(a + b \cdot t^2)}{dt} = \frac{d(a)}{dt} + \frac{d(bt^2)}{dt} \\
= 0 + \frac{d(bt^2)}{dt} = b \cdot \frac{d(t^2)}{dt} = 2b \cdot t
\]

(C.2')

(C.4)

(C.2)

(C.5)
C. Review of Calculus

Examples: $a,b$ are constants

example (e.2): $f(t) = a \cdot \sin(b \cdot t)$

$$\frac{df(t)}{dt} = \frac{d(a \sin(b \cdot t))}{dt} = a \frac{d(\sin(b \cdot t))}{dt}$$

$$= a \cdot \cos(b \cdot t) \cdot \frac{d(b \cdot t)}{dt} = a \cdot b \cdot \cos(bt) \cdot \frac{d(t)}{dt}$$

$$= ab \cos(bt)$$
C. Review of Calculus

Examples: $a, b$ are constants

example (e.3): $f(t) = a \cdot e^{(-bt)}$

$$\frac{df(t)}{dt} = \frac{d(a \cdot e^{(-bt)})}{dt} = a \frac{d(e^{(-bt)})}{dt}$$

(C.2)

$$= a \cdot e^{(-bt)} \frac{d(-bt)}{dt}$$

(C.8)(C.3)

$$= -ab \cdot e^{(-bt)}$$

(C.2)(C.5)

Integral of $f(t)$: not needed in this course