Question 1. (30 Points) A skier has found a nice Christmas tree at the top of a snow-covered mountain. She cuts it down and ties it to herself using a rope and proceeds to ski down the mountain. The coefficient of kinetic friction between her skis and the slope can be considered to be 0, but between the tree and the mountain, the coefficient of kinetic friction is $\mu_t = 0.2$.

A. (10 points) Derive an expression for the acceleration of the skier down the slope in terms of the mass of the skier $m_s$, the mass of the tree $m_t$, the acceleration due to gravity $g$, the angle of inclination of the mountain $\theta$, and the coefficient of friction of the tree $\mu_t$.

B. (10 points) If the skier’s mass (with skis, poles, boots, etc.) is 80kg and the tree’s mass is 400kg, for which angle $\theta$ is the skier’s velocity down the mountain constant?

C. (10 points) When the velocity down the slope is constant, what is the tension in the rope joining the skier and the tree?
Question 2. (35 points) A freedom loving American in the back of a stationary pickup truck fires his rifle in to the air at an angle $\theta$ to the horizontal. Assume that the bullet is not subject to drag forces. At the same time as the gun is fired the driver of the pickup truck begins to accelerate the truck to the right with a constant acceleration $a$.

A. (5 points) If after 10 s the pickup truck is 250 m down the road from where it started, what is the value of the acceleration $a$?

B. (5 points) How fast is the pickup truck going at this point?

C. (10 points) At this time the freedom loving American is hit by the bullet he fired into the air. What is the initial velocity of the bullet? Express your answer using unit vector notation, using $\hat{i}$ for the direction the truck moves in and $\hat{j}$ for vertically upward.

D. (5 points) What is the angle $\theta$ at which the rifle is fired?

E. (5 points) What is the maximum height that the bullet reaches before it turns around?

F. (5 points) If the bullet fired has mass 100 g find the value of it’s total mechanical energy at 3 positions: when it is first fired, when it is at the top of it’s trajectory, and just before it hits the freedom loving American.
Question 3. (35 points)

To get around the world quickly Santa relies on high speed circular orbits around the Earth, in which the centripetal force is provided by gravity. (The initial acceleration to get him to the required velocity for this orbit is provided by Rudolf).

(a) (15 points) If he wants to complete a full orbit in 2 hrs at what height above the surface of the earth must he fly?

(b) (10 points) At what speed is Santa moving while he is in this orbit?

(c) (10 points) Santa has 1000 kg of presents in his sleigh. What is the difference in the gravitational potential energy for those presents when they are in orbit compared to when they are on the ground?